

# Analysis and investigation of inductively coupled coils in linear electric circuits

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**Abstract.** This paper presents a comprehensive investigation of linear electric circuits with inductively coupled coils, focusing on the analysis of mutual inductance ( $M$ ) and the coupling coefficient ( $k$ ). The study begins with a theoretical overview, including the fundamental principles of electromagnetic induction and the mathematical relationships describing the behavior of inductively coupled coils. Experimental setups were used to explore the influence of key parameters, such as the distance between the coils and the angle between their planes, on the values of  $M$  and  $k$ . The results demonstrate a clear dependence of mutual inductance and coupling coefficient on these parameters, with mutual inductance decreasing as the distance between the coils increases or as the angle between their planes approaches  $90^\circ$ . The measured data are combined with mathematical calculations to validate the accuracy of the experiments. The conclusions drawn from this study provide useful tips into the optimization of inductively coupled coil systems, which are widely used in power transmission, signal processing, wireless energy transfer, and electromagnetic sensor applications. These findings can be applied in the design and analysis of efficient electromagnetic devices.

**Keywords:** circuit, coils, coupling, electric, energy, inductance, magnetic

## 1 Introduction

This paper investigates the characteristics of linear electric circuits with inductively coupled coils, providing a detailed analysis of the mutual inductance and coupling coefficient, supported by experimental data.

Inductively coupled coils play a crucial role in the transmission of energy and signals in electrical systems. These components rely on the magnetic field generated by current flowing through one coil to induce a voltage in another (Армянов & Стоянова, 2001; Генов, 1991; Савов, 2006; Станев, 2008). The extent of this interaction is measured by the mutual inductance, which depends on the geometry of the coils, the number of turns, and the magnetic permeability of the medium. This concept is rooted in the electromagnetic induction principle, discovered by Michael Faraday, and is widely applied in practice. The induced voltage in the secondary coil is given by (Фархи & Папазов, 1989)

$$U_2 = -M \frac{dI_1}{dt} \quad (1)$$

where  $M$  is the mutual inductance,  $I_1$  is the current through the primary coil, and  $U_2$  is the induced voltage in the secondary coil.

Mutual inductance  $M$  is a function of the inductances of the two coils  $L_1$  and  $L_2$ , and the coupling coefficient  $k$ , which represents the degree of magnetic linkage between them. The relationship between these quantities is expressed as (Sebastián et al., 2020)

$$M = k \cdot \sqrt{L_1 \cdot L_2} \quad (2)$$

The coupling coefficient ranges between 0 (no coupling) and 1 (perfect coupling), with its value depending on the distance between the coils and their orientation.

The analysis of circuits with inductively coupled coils requires the use of Kirchhoff's laws and Faraday's law of induction. For two coils with mutual inductance, the general equations for the voltages and currents are expressed as (Фархи & Папазов, 1989)

$$U_1 = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} \quad (3)$$

and

$$U_2 = L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt} \quad (4)$$

These equations describe the dynamics of the coupled coils based on the changes in currents and inductances.

When analysing circuits with inductively coupled coils, it is important to consider the polarity of the induced voltage, which depends on the winding direction and the coupling between the coils. A common notation is used to indicate the relative orientation of the currents in both coils. Equivalent circuit models are often used to simplify the analysis of complex circuits.

By transforming inductively coupled coils into equivalent transformers, the analysis of circuits involving mutual inductance can be simplified through techniques such as equivalent transformer circuits and the method of reflected impedance.

Inductively coupled coils are widely used in various electronic and electrical systems. Transformers, which facilitate the transfer of electrical energy, are the most common example. In these devices, the turns ratio between the primary and secondary coils determines the voltage and current in the secondary circuit. Other applications include radio frequency filters, where inductively coupled coils are used to tune the circuit's resonance frequency, and oscillator circuits, where they provide feedback, to sustain oscillations.

Mutual inductance of two inductively coupled coils can be determined experimentally by connecting only one coil to a sinusoidal voltage source and experimentally measuring the excitation current through it and the open-circuit voltage at the terminals of the other coil (Савов и др., 2010)

$$M = \frac{U_{M21}}{\omega I_1} = \frac{U_{M12}}{\omega I_2} \quad (5)$$

The degree of magnetic coupling between two inductively coupled coils is expressed by the coupling coefficient  $k$ , defined as the geometric mean of the relative values of the mutual induction magnetic fluxes in relation to the self-induction fluxes (Abu-Saude & Bashir, 2023; Sari, 2024)

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}} \quad (6)$$

## 2 Experiments

This paper presents an experimental model consisting of two inductively coupled coils, designed to demonstrate the principles of electromagnetic induction. When an electric current flows through one coil, it generates a magnetic field that induces a voltage in the other coil, without requiring any physical connection. This model serves as an effective tool for studying the interaction between electrical circuits and magnetic fields.

The experiments were conducted at the Technical University of Varna, Bulgaria. All measurements and observations were performed in laboratory 306E within the Department of Electrical Engineering. The measuring devices and other required equipment were provided by the same laboratory.

## 2.1 Experiment one

The schematic of the experimental setup and a photograph of the experimental model are shown in Figures 1 and 2, respectively.

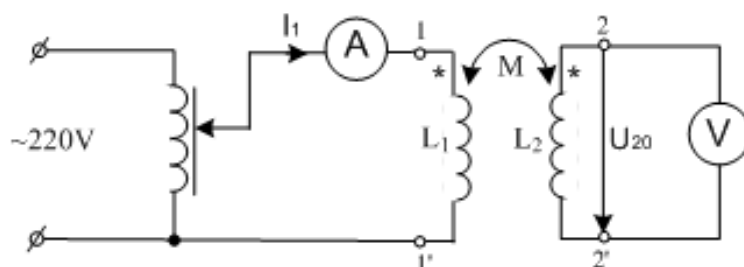


Fig. 1. Experimental scheme.

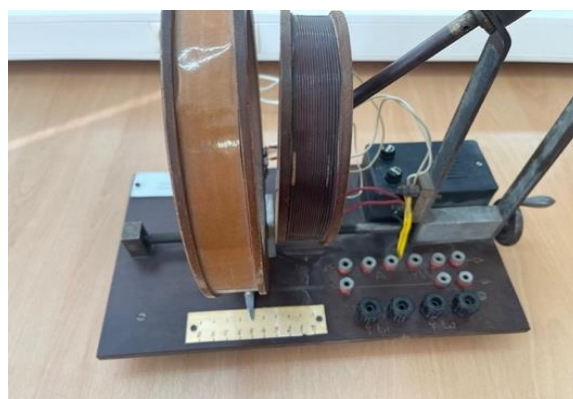


Fig. 2. Experimental model with two inductively coupled coils.

For the electrical circuit diagram depicted in Fig. 1, determine the mutual inductance  $M$  and the coupling coefficient  $k$  by measuring the open-circuit induced voltage of the second coil  $U_{20} = U_{M21}$ . Investigate the variation of  $M$  and  $k$  as a function of the distance  $l$  between coils, while maintaining a constant angle between their planes  $\beta = 0$  (Панов и др., 2017).

The excitation voltage applied to the primary coil is supplied via an autotransformer. The current in the circuit is monitored using an ammeter and has a measured value of  $I_1 = 0,47$  A. The induced voltage across the secondary coil is recorded with a voltmeter to evaluate the magnetic coupling and energy transfer characteristics between the coils. The values of the primary and secondary coils are  $L_1 = 11,8$  mH  $L_2 = 7,2$  mH.

### 2.1.1 Result

Mutual inductance  $M$  and coupling coefficient  $k$  were evaluated based on the analytical expressions provided in equations (5) and (6), respectively. The results are shown in Table 1 and illustrated in the graph below.

Table 1. Results from experiment one.

$l$	cm	1	2	3	5
$U_{20}$	V	0,7	0,6	0,55	0,4
$M$	H	0,005	0,004	0,0035	0,003
$k$	-	0,5	0,4	0,35	0,3

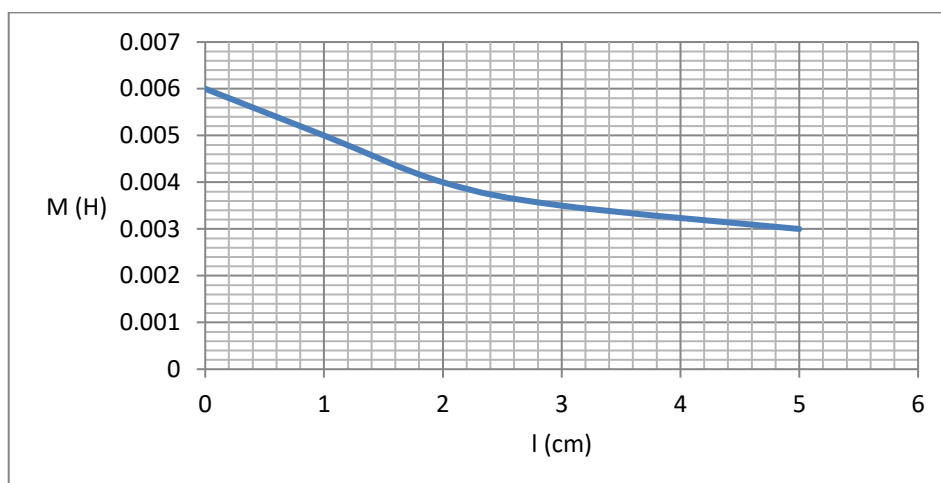


Fig. 3. Graph of dependency  $M = f(l)$ .

## 2.2 Experiment two

The schematic of the experimental setup and a photograph of the experimental model are shown in Figures 4 and 5. For the electrical circuit diagram shown in Fig. 4, determine the mutual inductance  $M$  and the coupling coefficient  $k$  by measuring the open-circuit induced voltage of the second coil  $U_{20} = U_{M21}$ . Investigate the variation of  $M$  and  $k$  as a function of altering the angle between their planes  $\beta$  while maintaining a constant distance  $l = 0$  (Савов и др., 2010).

The first coil  $L_1$  is powered using an autotransformer. The current flowing through the circuit is measured by an ammeter and has a value of  $I_1 = 0,47$  A. The voltage induced across the second coil  $L_2$  is determined using a voltmeter.

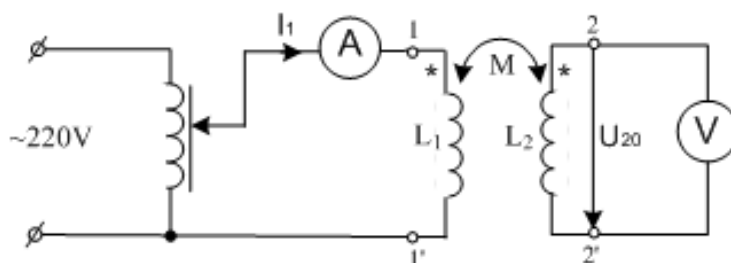


Fig. 4. Experimental scheme.

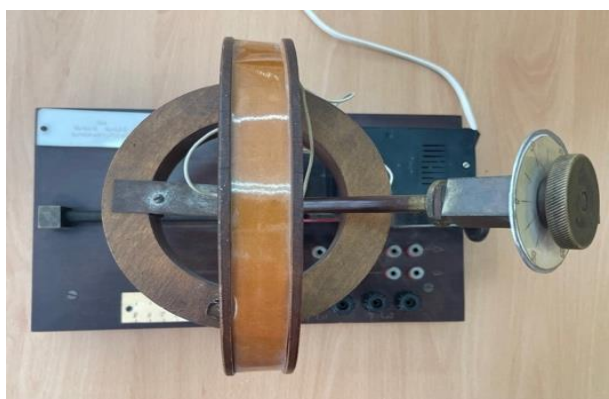


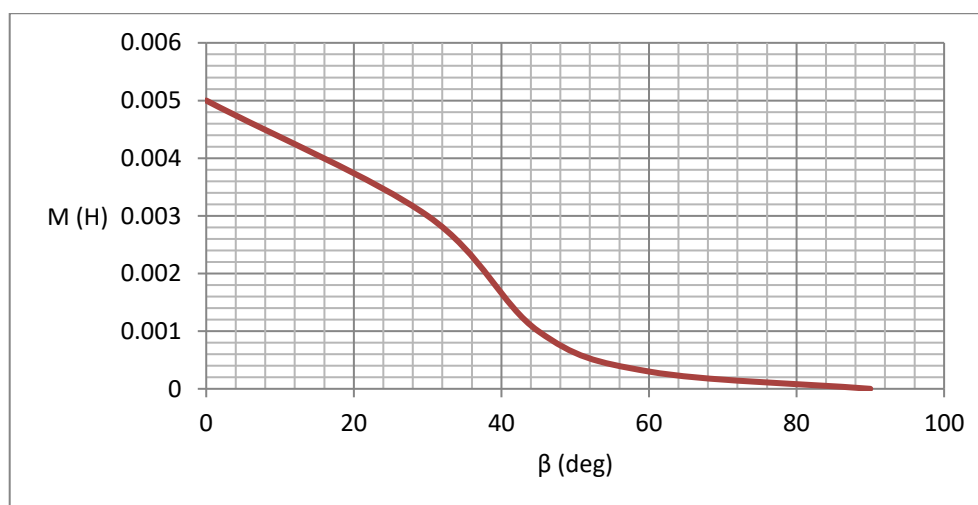
Fig. 5. Experimental model with two inductively coupled coils.

### 2.2.1 Results

The values of mutual inductance  $M$  and coupling coefficient  $k$  were determined using equations (5) and (6), respectively. The results are shown in Table 2 and illustrated in the graph below.

**Table 2.** Results from experiment two.

$\beta$	$^\circ$	0	30	45	60	90
$U_{20}$	V	0,7	0,4	0,2	0,05	0
$M$	H	0,005	0,003	0,001	0,0003	0
$k$	-	0,5	0,3	0,1	0,03	0



**Fig. 6.** Graph of dependency  $M = f(\beta)$ .

## 3 Conclusion

Inductively coupled coils are central to many fields of electrical engineering and electronics. Their ability to induce voltage through magnetic linkage makes them indispensable in applications ranging from energy transmission to signal filtering. Understanding their theory and analysis methods is essential for the efficient design of electrical and electronic systems.

The experimental model used in this research proved to be an effective tool for studying the interaction between electrical circuits and magnetic fields, without requiring any physical connection between the coils. This highlights the importance of such models for educational and research purposes in the field of electromagnetism.

The conducted experiments and analysis have highlighted the critical factors influencing the performance of inductively coupled coils in linear electric circuits. The results demonstrate that the mutual inductance and coupling coefficient are strongly dependent on the relative position and orientation of the coils.

The following important conclusions can be drawn from the research:

From Experiment one, it can be summarized that as the distance between the coils increases,  $M$  and  $k$  decrease. This is evidenced by the results obtained in Table 1 and presented in Figure 3.

In Experiment two, the results presented in Table 2 and Figure 6 demonstrate that as the angle between the coils increases from 0 to 90 degrees, the mutual inductance and coupling coefficient decrease. At 90 degrees, they become equal to zero. This phenomenon allows for the limitation of the magnetic field effect emitted from the coils without shielding them. This finding has significant implications for the construction and utilization of various electromagnetic devices.

These results are consistent with the theoretical principles of electromagnetic induction and provide a clear understanding of how to control inductive coupling in practical applications.

In conclusion, this paper delineates the fundamental aspects of the issue under consideration and can be further developed with additional experiments, simulations, or computational models to address the requirements of diverse tasks.

Future work may focus on extending the analysis to non-linear circuits, exploring the effects of different core materials, or investigating the impact of external magnetic fields on the performance of inductively coupled coils.

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